

Algebraic Number Theory

(PARI-GP version 2.17.1)

Binary Quadratic Forms

create $ax^2 + bxy + cy^2$ **Qfb**(a, b, c) or **Qfb**($[a, b, c]$)
 reduce x ($s = \sqrt{D}$, $l = \lfloor s \rfloor$) **qfbred**($x, \{flag\}, \{D\}, \{l\}, \{s\}$)
 return $[y, g]$, $g \in \text{SL}_2(\mathbf{Z})$, $y = g \cdot x$ reduced **qfbreds12**(x)
 composition of forms $x*y$ or **qfbnucomp**(x, y, l)
 n -th power of form x^n or **qfbnupow**(x, n)
 composition **qfbcomp**(x, y)
 ... without reduction **qfbcomprow**(x, y)
 n -th power **qfbpow**(x, n)
 ... without reduction **qfbpowrow**(x, n)
 prime form of disc. x above prime p **qfbprimeform**(x, p)
 class number of disc. x **qfbclassno**(x)
 Hurwitz class number of disc. x **qfbhclassno**(x)
 solve $Q(x, y) = n$ in integers **qfbsolve**(Q, n)
 solve $x^2 + Dy^2 = p$, p prime **qfbcornacchia**(D, p)
 ... $x^2 + Dy^2 = 4p$, p prime **qfbcornacchia**($D, 4 * p$)

Quadratic Fields

quadratic number $\omega = \sqrt{x}$ or $(1 + \sqrt{x})/2$ **quadgen**(x)
 minimal polynomial of ω **quadpoly**(x)
 discriminant of $\mathbf{Q}(\sqrt{x})$ **quaddisc**(x)
 regulator of real quadratic field **quadregulator**(x)
 fundamental unit in O_D , $D > 0$ **quadunit**($D, \{t\}$)
 norm of fundamental unit in O_D **quadunitnorm**(D)
 index of $O_{Df^2}^\times$ in O_D^\times **quadunitindex**(D, f)
 class group of $\mathbf{Q}(\sqrt{D})$ **quadclassunit**($D, \{flag\}, \{t\}$)
 Hilbert class field of $\mathbf{Q}(\sqrt{D})$ **quadhilbert**($D, \{flag\}$)
 ... using specific class invariant ($D < 0$) **polclass**($D, \{inv\}$)
 test if T is **polclass**(D); if so return D **polisclass**(T)
 ray class field modulo f of $\mathbf{Q}(\sqrt{D})$ **quadray**($D, f, \{flag\}$)

General Number Fields: Initializations

The number field $K = \mathbf{Q}[X]/(f)$ is given by irreducible $f \in \mathbf{Q}[X]$.
 We denote $\theta = \bar{X}$ the canonical root of f in K . A nf structure
 contains a maximal order and allows operations on elements and
 ideals. A bnf adds class group and units. A bnr is attached to ray
 class groups and class field theory. A rmf is attached to relative
 extensions L/K .

init number field structure nf **nfinit**($f, \{flag\}$)
 known integer basis B **nfinit**($[f, B]$)
 order maximal at $vp = [p_1, \dots, p_k]$ **nfinit**($[f, vp]$)
 order maximal at all $p \leq P$ **nfinit**($[f, P]$)
 certify maximal order **nfcertify**(nf)

nf members:

a monic $F \in \mathbf{Z}[X]$ defining K **nf.pol**
 number of real/complex places **nf.r1/r2/sign**
 discriminant of nf **nf.disc**
 primes ramified in nf **nf.p**
 T_2 matrix **nf.t2**
 complex roots of F **nf.roots**
 integral basis of \mathbf{Z}_K as powers of θ **nf.zk**
 different/codifferent **nf.diff**, **nf.codiff**
 index $[\mathbf{Z}_K : \mathbf{Z}[X]/(F)]$ **nf.index**
 recompute nf using current precision **nfnewprec**(nf)
 init relative rmf $L = K[Y]/(g)$ **rnfinit**(nf, g)
 init bnf structure **bnfinit**($f, 1$)

bnf members: same as nf , plus

underlying nf
 class group, regulator
 fundamental/torsion units
 add S -class group and units, yield $bnfS$
 init class field structure bnr
bnr members: same as bnf , plus
 underlying bnf
 big ideal structure
 modulus m
 structure of $(\mathbf{Z}_K/m)^*$

Fields, subfields, embeddings

Defining polynomials, embeddings

(some) number fields with Galois group G **nflist**(G)
 ... and $|\text{disc}(K)| = N$ and s complex places **nflist**($G, N, \{s\}$)
 ... and $a \leq |\text{disc}(K)| \leq b$ **nflist**($G, [a, b], \{s\}$)
 smallest poly defining $f = 0$ (slow) **polredabs**($f, \{flag\}$)
 small poly defining $f = 0$ (fast) **polredbest**($f, \{flag\}$)
 monic integral $g = Cf(x/L)$ **poltomonic**($f, \{\&L\}$)
 random Tschirnhausen transform of f **poltschirnhaus**(f)
 $\mathbf{Q}[t]/(f) \subset \mathbf{Q}[t]/(g)$? Isomorphic? **nfisincl**(f, g), **nfisisom**
 reverse polmod $a = A(t) \bmod T(t)$ **modreverse**(a)
 compositum of $\mathbf{Q}[t]/(f)$, $\mathbf{Q}[t]/(g)$ **polcompositum**($f, g, \{flag\}$)
 compositum of $K[t]/(f)$, $K[t]/(g)$ **nfcompositum**($nf, f, g, \{flag\}$)
 splitting field of K (degree divides d) **nfsplitting**($nf, \{d\}$)
 signs of real embeddings of x **nfeltsign**($nf, x, \{pl\}$)
 complex embeddings of x **nfeltembed**($nf, x, \{pl\}$)
 $T \in K[t]$, $\#$ of real roots of $\sigma(T) \in R[t]$ **nfpolsturm**($nf, T, \{pl\}$)
 absolute Weil height **nfweilheight**(nf, v)

Subfields, polynomial factorization

subfields (of degree d) of nf **nfsubfields**($nf, \{d\}$)
 maximal subfields of nf **nfsubfieldsmax**(nf)
 maximal CM subfield of nf **nfsubfieldscm**(nf)
 $K_d \subset \mathbf{Q}(\zeta_n)$, using Gaussian periods **polsubcyclo**($n, d, \{v\}$)
 ... using class field theory **polsubcyclofast**(n, d)
 roots of unity in nf **nfroots0f1**(nf)
 roots of g belonging to nf **nfroots**(nf, g)
 factor g in nf **nfactor**(nf, g)

Linear and algebraic relations

poly of degree $\leq k$ with root $x \in \mathbf{C}$ or \mathbf{Q}_p **algdep**(x, k)
 alg. dep. with pol. coeffs for series s **seralgdep**(s, x, y)
 diff. dep. with pol. coeffs for series s **serdiffdep**(s, x, y)
 small linear rel. on coords of vector x **lindep**(x)

Basic Number Field Arithmetic (nf)

Number field elements are **t_INT**, **t_FRAC**, **t_POL**, **t_POLMOD**, or **t_COL**
 (on integral basis $nf.zk$).

Basic operations

$x + y$ **nfeltadd**(nf, x, y)
 $x \times y$ **nfeltmul**(nf, x, y)
 x^n , $n \in \mathbf{Z}$ **nfeltpow**(nf, x, n)
 x/y **nfeltdiv**(nf, x, y)
 $q = x \setminus y := \text{round}(x/y)$ **nfeltdivu**(nf, x, y)
 $r = x \% y := x - (x \setminus y)y$ **nfeltmod**(nf, x, y)
 ... $[q, r]$ as above **nfeltdivrem**(nf, x, y)
 reduce x modulo ideal A **nfeltreduce**(nf, x, A)
 absolute trace $\text{Tr}_{K/\mathbf{Q}}(x)$ **nfelttrace**(nf, x)
 absolute norm $N_{K/\mathbf{Q}}(x)$ **nfeltnorm**(nf, x)

is x a square? **nfeltissquare**($nf, x, \{\&y\}$)
 ... an n -th power? **nfeltispower**($nf, x, n, \{\&y\}$)

Multiplicative structure of K^* ; $K^*/(K^*)^n$

valuation $v_{\mathfrak{p}}(x)$ **nfeltval**(nf, x, \mathfrak{p})
 ... write $x = \pi^{v_{\mathfrak{p}}(x)}y$ **nfeltval**($nf, x, \mathfrak{p}, \&y$)
 quadratic Hilbert symbol (at \mathfrak{p}) **nfhilbert**($nf, a, b, \{\mathfrak{p}\}$)
 b such that $xb^n = v$ is small **idealredmodpower**(nf, x, n)

Maximal order and discriminant

integral basis of field $\mathbf{Q}[x]/(f)$ **nfbasis**(f)
 field discriminant of $\mathbf{Q}[x]/(f)$ **nfdisc**(f)
 ... and factorization **nfdiscfactors**(f)
 express x on integer basis **nfalgtobasis**(nf, x)
 express element x as a polmod **nfbasistoalg**(nf, x)

Hecke Grossencharacters

Let K be a number field and m a modulus. A $gchar$ structure
 describes the group of Hecke Grossencharacters of K of modulus m
 and allows computations with these characters. A character χ is
 described by its components modulo $gc.cyc$.

init $gchar$ structure gc for modulus m **gcharinit**($bnf, m, \{cm\}$)

gc members:

underlying bnf **gc.bnf**
 modulus **gc.mod**
 elementary divisors (including 0s) **gc.cyc**
 recompute gc using current precision **gcharnewprec**(gc)
 evaluate Hecke character chi at ideal id **gchareval**(gc, chi, id)
 exponent column of id in \mathbf{R}^n **gcharideallog**(gc, id)
 log representation of ideal id **gcharlog**(gc, id)
 ... of character χ **gcharallog**(gc, chi)
 exponent vector of χ in \mathbf{R}^n **gcharparameters**(gc, chi)
 conductor of χ **gcharconductor**(gc, chi)
 L-function of χ **lfunccreate**($[gc, chi]$)
 local component χ_v of χ **gcharlocal**(gc, chi, v)
 χ s.t. $\chi_v \approx Lchiv[i]$ for $v = Lv[i]$ **gcharidentify**($gc, Lv, Lchiv$)
 basis of group of algebraic characters **gcharalgebraic**(gc)
 algebraic character of given infinity type **gcharalgebraic**($gc, type$)
 is χ algebraic? **gcharisalgebraic**(gc, chi)

Dedekind Zeta Function ζ_K , Hecke L series

$R = [c, w, h]$ in initialization means we restrict $s \in \mathbf{C}$ to domain
 $|\Re(s) - c| < w$, $|\Im(s)| < h$; $R = [w, h]$ encodes $[1/2, w, h]$ and $[h]$
 encodes $R = [1/2, 0, h]$ (critical line up to height h).

ζ_K as Dirichlet series, $N(I) \leq b$ **dirzetak**(nf, b)
 init $\zeta_K^{(k)}(s)$ for $k \leq n$ **L = lfunit**($bnf, R, \{n = 0\}$)
 compute $\zeta_K(s)$ (n -th derivative) **lfun**($L, s, \{n = 0\}$)
 compute $\Lambda_K(s)$ (n -th derivative) **lfunlambda**($L, s, \{n = 0\}$)

init $L_K^{(k)}(s, \chi)$ for $k \leq n$ **L = lfunit**($[bnr, chi], R, \{n = 0\}$)
 compute $L_K(s, \chi)$ (n -th derivative) **lfun**($L, s, \{n\}$)
 Artin root number of K **bnrrootnumber**($bnr, chi, \{flag\}$)
 $L(1, \chi)$, for all χ trivial on H **bnrL1**($bnr, \{H\}, \{flag\}$)

Class Groups & Units (bnf, bnr)

Class field theory data $a_1, \{a_2\}$ is usually bnr (ray class field),
 bnr, H (congruence subgroup) or bnr, χ (character on **bnr.clgp**).
 Any of these define a unique abelian extension of K .

units / S -units **bnfunits**($bnf, \{S\}$)
 remove GRH assumption from bnf **bnfcertify**(bnf)

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expo. of ideal x on class gp `bnfisprincipal(bnf, x, {flag})`
 ... on ray class gp `bnrisprincipal(bnr, x, {flag})`
 expo. of x on fund. units `bnfisunit(bnf, x)`
 ... on S -units, U is `bnfunits(bnf, S)` `bnfisunit(bnfs, x, U)`
 signs of real embeddings of $bnf.fu$ `bnfsignunit(bnf)`
 narrow class group `bnfnarrow(bnf)`

Class Field Theory

ray class number for modulus m `bnrclassno(bnf, m)`
 discriminant of class field `bnrdisc(a1, {a2})`
 ray class numbers, l list of moduli `bnrclasslist(bnf, l)`
 discriminants of class fields `bnrdisclist(bnf, l, {arch}, {flag})`
 decode output from `bnrdisclist` `bnfdecodemodule(nf, fa)`
 is modulus the conductor? `bnrisconductor(a1, {a2})`
 is class field (bnr, H) Galois over K^G `bnrisgalois(bnr, G, H)`
 action of automorphism on `bnr.gen` `bnrgaloismatrix(bnr, aut)`
 apply `bnrgaloismatrix M` to H `bnrgaloisapply(bnr, M, H)`
 characters on `bnr.clgp` s.t. $\chi(g_i) = e(v_i)$ `bnrchar(bnr, g, {v})`
 conductor of character χ `bnrconductor(bnr, chi)`
 conductor of extension `bnrconductor(a1, {a2}, {flag})`
 conductor of extension $K[Y]/(g)$ `rnfconductor(bnf, g)`
 canonical projection $Cl_F \rightarrow Cl_f, f | F$ `bnrmap`
 Artin group of extension $K[Y]/(g)$ `rnfnormgroup(bnr, g)`
 subgroups of bnr , index $\leq b$ `subgrouplist(bnr, b, {flag})`
 compositum as `[bnr, H]` `bnrcompositum([bnr1, H1], [bnr2, H2])`
 class field defined by $H < Cl_f$ `bnrclassfield(bnr, H)`
 ... low level equivalent, prime degree `rnfkummer(bnr, H)`
 same, using Stark units (real field) `bnrstark(bnr, {sub}, {flag})`
 Stark unit `bnrstarkunit(bnr, {sub})`
 is a an n -th power in K_v ? `nfislocalpower(nf, v, a, n)`
 cyclic L/K satisf. local conditions `nfgrunwaldwang(nf, P, D, pl)`

Cyclotomic and Abelian fields theory

An Abelian field F given by a subgroup $H < (Z/fZ)^*$ is described by an argument F , e.g. f (for $H = 1$, i.e. $Q(\zeta_f)$) or $[G, H]$, where G is `idealstar(f, 1)`, or a minimal polynomial.
 minus class number $h^-(F)$ `subcyclohminus(F)`
 ... p -part `subcyclohminus(F, p)`
 minus part of Iwasawa polynomials `subcycloiwawasawa(F, p)`
 p -Sylow of $Cl(F)$ `subcycloplgp(F, p)`

Logarithmic class group

logarithmic ℓ -class group `bnflog(bnf, \ell)`
 $[\tilde{e}(F_v/Q_p), \tilde{f}(F_v/Q_p)]$ `bnfloget(bnf, pr)`
 exp deg $F(A)$ `bnflogdegree(bnf, A, \ell)`
 is ℓ -extension L/K locally cyclotomic `rnfislocalcyclo(rnf)`

Ideals: elements, primes, or matrix of generators in HNF

is id an ideal in nf ? `nfisideal(nf, id)`
 is x principal in bnf ? `bnfisprincipal(bnf, x)`
 give $[a, b]$, s.t. $aZ_K + bZ_K = x$ `idealtwoelt(nf, x, {a})`
 put ideal $a(aZ_K + bZ_K)$ in HNF form `idealhnf(nf, a, {b})`
 norm of ideal x `idealnrm(nf, x)`
 minimum of ideal x (direction v) `ideallmin(nf, x, v)`
 LLL-reduce the ideal x (direction v) `idealred(nf, x, {v})`

Ideal Operations

add ideals x and y `idealadd(nf, x, y)`
 multiply ideals x and y `idealmul(nf, x, y, {flag})`
 intersection of ideal x with Q `idealdn(nf, x)`
 intersection of ideals x and y `idealintersect(nf, x, y, {flag})`
 n -th power of ideal x `idealpow(nf, x, n, {flag})`
 inverse of ideal x `idealinvt(nf, x)`

divide ideal x by y `idealdiv(nf, x, y, {flag})`
 Find $(a, b) \in x \times y, a + b = 1$ `idealaddtoone(nf, x, {y})`
 coprime integral A, B such that $x = A/B$ `idealnumden(nf, x)`

Primes and Multiplicative Structure

check whether x is a maximal ideal `idealismaximal(nf, x)`
 factor ideal x in Z_K `idealfactor(nf, x)`
 expand ideal factorization in K `idealfactorback(nf, f, {e})`
 is ideal A an n -th power? `idealispower(nf, A, n)`
 expand elt factorization in K `nffactorback(nf, f, {e})`
 decomposition of prime p in Z_K `idealprimedec(nf, p)`
 valuation of x at prime ideal pr `idealval(nf, x, pr)`
 weak approximation theorem in nf `idealchinese(nf, x, y)`
 $a \in K$, s.t. $v_p(a) = v_p(x)$ if $v_p(x) \neq 0$ `idealappr(nf, x)`
 $a \in K$ such that $(a \cdot x, y) = 1$ `idealcoprime(nf, x, y)`
 give $bid = \text{structure of } (Z_K/id)^*$ `idealstar(nf, id, {flag})`
 structure of $(1 + \mathfrak{p})/(1 + \mathfrak{p}^k)$ `idealprincipalunits(nf, pr, k)`
 discrete log of x in $(Z_K/bid)^*$ `ideallog(nf, x, bid)`
`idealstar` of all ideals of norm $\leq b$ `ideallist(nf, b, {flag})`
 add Archimedean places `ideallistarch(nf, b, {ar}, {flag})`
 init `modpr` structure `nfmodprinit(nf, pr, {v})`
 project t to Z_K/pr `nfmodpr(nf, t, modpr)`
 lift from Z_K/pr `nfmodprlift(nf, t, modpr)`

Galois theory over Q

conjugates of a root θ of nf `nfgaloisconj(nf, {flag})`
 apply Galois automorphism s to x `nfgaloisapply(nf, s, x)`
 Galois group of field $Q[x]/(f)$ `polgalois(f)`
 resolvent field of $Q[x]/(f)$ `nfresolvent(f)`
 initializes a Galois group structure G `galoisinit(pol, {den})`
 ... for the splitting field of pol `galoissplittinginit(pol, {d})`
 character table of G `galoischartable(G)`
 conjugacy classes of G `galoisconjclasses(G)`
 $\det(1 - \rho(g)T)$, χ character of ρ `galoischarpoly(G, \chi, {o})`
 $\det(\rho(g))$, χ character of ρ `galoischarpoly(G, \chi, {o})`
 action of p in `nfgaloisconj` form `galoispermtpol(G, {p})`
 identify as abstract group `galoisidentify(G)`
 export a group for GAP/MAGMA `galoisexport(G, {flag})`
 subgroups of the Galois group G `galoissubgroups(G)`
 is subgroup H normal? `galoisisnormal(G, H)`
 subfields from subgroups `galoissubfields(G, {flag}, {v})`
 fixed field `galoisfixedfield(G, perm, {flag}, {v})`
 Frobenius at maximal ideal P `idealfrobenius(nf, G, P)`
 ramification groups at P `idealramgroups(nf, G, P)`
 is G abelian? `galoisisabelian(G, {flag})`
 abelian number fields/ Q `galoissubcyclo(N, H, {flag}, {v})`

The galpol package

query the package: polynomial `galoisgetpol(a, b, {s})`
 ...: permutation group `galoisgetgroup(a, b)`
 ...: group description `galoisgetname(a, b)`

Relative Number Fields (rnf)

Extension L/K is defined by $T \in K[x]$.
 absolute equation of L `rnfequation(nf, T, {flag})`
 is L/K abelian? `rnfisabelian(nf, T)`
 relative `nfgalgtobasis` `rnfalgtoabasis(rnf, x)`
 relative `nfbasistoalg` `rnfbasistoalg(rnf, x)`
 relative `idealhnf` `rnfidealhnf(rnf, x)`

relative `idealmul` `rnfidealmul(rnf, x, y)`
 relative `idealtwoelt` `rnfidealtwoelt(rnf, x)`

Lifts and Push-downs

absolute \rightarrow relative representation for x `rnfeltabstorel(rnf, x)`
 relative \rightarrow absolute representation for x `rnfeltreltoabs(rnf, x)`
 lift x to the relative field `rnfeltup(rnf, x)`
 push x down to the base field `rnfeltdown(rnf, x)`
 idem for x ideal: `(rnfideal)reltoabs, abstorel, up, down`

Norms and Trace

relative norm of element $x \in L$ `rnfeltnorm(rnf, x)`
 relative trace of element $x \in L$ `rnfeltrtrace(rnf, x)`
 absolute norm of ideal x `rnfidealnormabs(rnf, x)`
 relative norm of ideal x `rnfidealnormrel(rnf, x)`
 solutions of $N_{K/Q}(y) = x \in Z$ `bnfisintnorm(bnf, x)`
 is $x \in Q$ a norm from K ? `bnfisnorm(bnf, x, {flag})`
 initialize T for norm eq. solver `rnfnisnorminit(K, pol, {flag})`
 is $a \in K$ a norm from L ? `rnfnisnorm(T, a, {flag})`
 initialize t for Thue equation solver `thueinit(f)`
 solve Thue equation $f(x, y) = a$ `thue(t, a, {sol})`
 characteristic poly. of a mod T `rnfcharpoly(nf, T, a, {v})`

Factorization

factor ideal x in L `rnfidealfactor(rnf, x)`
 $[S, T]: T_{i,j} | S_i; S$ primes of K above p `rnfidealprimedec(rnf, p)`

Maximal order Z_L as a Z_K -module

relative `polredbest` `rnfpolredbest(nf, T)`
 relative `polredabs` `rnfpolredabs(nf, T)`
 relative Dedekind criterion, prime pr `rnfdedekind(nf, T, pr)`
 discriminant of relative extension `rnfdisc(nf, T)`
 pseudo-basis of Z_L `rnfpsseudobasis(nf, T)`

General Z_K -modules: $M = [\text{matrix, vec. of ideals}] \subset L$
 relative HNF / SNF `nfhnf(nf, M), nfnfnf`
 multiple of det M `nfdetint(nf, M)`
 HNF of M where $d = nfdetint(M)$ `nfhnfmod(x, d)`
 reduced basis for M `rnfllgram(nf, T, M)`
 determinant of pseudo-matrix M `rnfdet(nf, M)`
 Steinitz class of M `rnfsteinitz(nf, M)`
 Z_K -basis of M if Z_K -free, or 0 `rnfhnfbasis(bnf, M)`
 n -basis of M , or $(n + 1)$ -generating set `rnfbasis(bnf, M)`
 is M a free Z_K -module? `rnfisfree(bnf, M)`

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Associative Algebras

A is a general associative algebra given by a multiplication table mt (over \mathbf{Q} or \mathbf{F}_p); represented by al from `algtableinit`.

create al from mt (over \mathbf{F}_p) `algtableinit(mt, {p = 0})`
 group algebra $\mathbf{Q}[G]$ (or $\mathbf{F}_p[G]$) `algggroup(G, {p = 0})`
 center of group algebra `alggrouppcenter(G, {p = 0})`

Properties

is (mt, p) OK for `algtableinit`? `algisassociative(mt, {p = 0})`
 multiplication table mt `algmtable(al)`
 dimension of A over prime subfield `algdim(al)`
 characteristic of A `algchar(al)`
 is A commutative? `algiscommutative(al)`
 is A simple? `algissimple(al)`
 is A semi-simple? `algissemisimple(al)`
 center of A `algcenter(al)`
 Jacobson radical of A `algradical(al)`
 radical J and simple factors of A/J `algsimpledec(al)`

Operations on algebras

create A/I , I two-sided ideal `algquotient(al, I)`
 create $A_1 \otimes A_2$ `algtensor(al1, al2)`
 create subalgebra from basis B `algsubalg(al, B)`
 quotients by ortho. central idempotents e `algcentralproj(al, e)`
 isomorphic alg. with integral mult. table `algmakeintegral(mt)`
 prime subalgebra of semi-simple A over \mathbf{F}_p `algprimesubalg(al)`
 find isomorphism $A \cong M_d(\mathbf{F}_q)$ `algsplit(al)`

Operations on lattices in algebras

lattice generated by cols. of M `alglathnf(al, M)`
 \dots by the products xy , $x \in lat1$, $y \in lat2$ `alglatmul(al, lat1, lat2)`
 sum $lat1 + lat2$ of the lattices `alglatadd(al, lat1, lat2)`
 intersection $lat1 \cap lat2$ `alglatinter(al, lat1, lat2)`
 test $lat1 \subset lat2$ `alglatsubset(al, lat1, lat2)`
 generalized index $(lat2 : lat1)$ `alglatindex(al, lat1, lat2)`
 $\{x \in al \mid x \cdot lat1 \subset lat2\}$ `alglatlefttransporter(al, lat1, lat2)`
 $\{x \in al \mid lat1 \cdot x \subset lat2\}$ `alglatrighttransporter(al, lat1, lat2)`
 test $x \in lat$ (set $c = \text{coord. of } x$) `alglatcontains(al, lat, x, {\&c})`
 element of lat with coordinates c `alglatelement(al, lat, c)`

Operations on elements

$a + b$, $a - b$, $-a$ `algadd(al, a, b)`, `algsub`, `algneg`
 $a \times b$, a^2 `algmul(al, a, b)`, `algsqr`
 a^n , a^{-1} `algpow(al, a, n)`, `alginv`
 is x invertible? (then set $z = x^{-1}$) `alginv(al, x, {\&z})`
 find z such that $x \times z = y$ `algdivl(al, x, y)`
 find z such that $z \times x = y$ `algdivr(al, x, y)`
 does z s.t. $x \times z = y$ exist? (set it) `algsdivl(al, x, y, {\&z})`
 matrix of $v \mapsto x \cdot v$ `algtomatrix(al, x)`
 absolute norm `algnorm(al, x)`
 absolute trace `algtrace(al, x)`
 absolute char. polynomial `algcharpoly(al, x)`
 given $a \in A$ and polynomial T , return $T(a)$ `algpoleval(al, T, a)`
 random element in a box `algrandom(al, b)`

Central Simple Algebras

A is a central simple algebra over a number field K ; represented by al from `alginit`; K is given by a nf structure.

create CSA from data `alginit(B, C, {v}, {maxord = 1})`
 multiplication table over K $B = K$, $C = mt$
 cyclic algebra $(L/K, \sigma, b)$ $B = rnf$, $C = [\sigma, b]$
 quaternion algebra $(a, b)_K$ $B = K$, $C = [a, b]$
 matrix algebra $M_d(K)$ $B = K$, $C = d$
 local Hasse invariants over K $B = K$, $C = [d, [PR, HF], HI]$

Properties

type of al (mt , CSA) `algtype(al)`
 dimension of A over \mathbf{Q} `algdim(al, 1)`
 dimension of al over its center K `algdim(al)`
 degree of A ($= \sqrt{\dim_K A}$) `algdegree(al)`
 al a cyclic algebra $(L/K, \sigma, b)$; return σ `algaut(al)`
 \dots return b `algb(al)`
 \dots return L/K , as an rnf `algsplittingfield(al)`
 split A over an extension of K `algsplittingdata(al)`
 splitting field of A as an rnf over center `algsplittingfield(al)`
 multiplication table over center `algrelmtable(al)`
 places of K at which A ramifies `algramifiedplaces(al)`
 Hasse invariants at finite places of K `alghassef(al)`
 Hasse invariants at infinite places of K `alghassei(al)`
 Hasse invariant at place v `alghasse(al, v)`
 index of A over K (at place v) `algindex(al, {v})`
 is al a division algebra? (at place v) `algsdivision(al, {v})`
 is A ramified? (at place v) `algsramified(al, {v})`
 is A split? (at place v) `algsisplit(al, {v})`

Operations on elements

reduced norm `algnorm(al, x)`
 reduced trace `algtrace(al, x)`
 reduced char. polynomial `algcharpoly(al, x)`
 express x on integral basis `algalgtobasis(al, x)`
 convert x to algebraic form `algbasistoalg(al, x)`
 map $x \in A$ to $M_d(L)$, L split. field `algtomatrix(al, x)`

Orders

\mathbf{Z} -basis of order \mathcal{O}_0 `algbasis(al)`
 discriminant of order \mathcal{O}_0 `algdisc(al)`
 \mathbf{Z} -basis of natural order in terms \mathcal{O}_0 's basis `alginvbasis(al)`

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