

# Pari-GP reference card

(PARI-GP version 2.18.0)

Note: optional arguments are surrounded by braces {}.  
To start the calculator, type its name in the terminal: **gp**  
To exit **gp**, type **quit**, **\q**, or **<C-D>** at prompt.

## Help

describe function *?function*  
extended description *??keyword*  
list of relevant help topics *???pattern*  
name of GP-1.39 function *f* in GP-2.\* *whatnow(f)*

## Input/Output

previous result, the result before *%, %~, %`*, etc.  
*n*-th result since startup *%n*  
separate multiple statements on line *;*  
extend statement on additional lines *\*  
extend statements on several lines *{seq1; seq2;}*  
comment */\* ... \*/*  
one-line comment, rest of line ignored *\\ ...*

## Metacommands & Defaults

set default *d* to *val* *default({d},{val})*  
toggle timer on/off *#*  
print time for last result *##*  
print defaults *\d*  
set debug level to *n* *\g n*  
set memory debug level to *n* *\gm n*  
set *n* significant digits / bits *\p n, \pb n*  
set *n* terms in series *\ps n*  
quit GP *\q*  
print the list of PARI types *\t*  
print the list of user-defined functions *\u*  
read file into GP *\r filename*  
set debuglevel for domain *D* to *n* *setdebug(D,n)*

## Debugger / break loop

get out of break loop *break* or **<C-D>**  
go up/down *n* frames *dbg\_up({n}), dbg\_down*  
set break point *breakpoint()*  
examine object *o* *dbg\_x(o)*  
current error data *dbg\_err()*  
number of objects on heap and their size *getheap()*  
total size of objects on PARI stack *getstack()*

## PARI Types & Input Formats

**t\_INT**. Integers; hex, binary *±31; ±0x1F, ±0b101*  
**t\_REAL**. Reals *±3.14, 6.022 E23*  
**t\_INTMOD**. Integers modulo *m* *Mod(n,m)*  
**t\_FRAC**. Rational Numbers *n/m*  
**t\_FFELT**. Elt in finite field **F<sub>q</sub>** *ffgen(q,'t)*  
**t\_COMPLEX**. Complex Numbers *x + y \* I*  
**t\_PADIC**. *p*-adic Numbers *x + 0(p^k)*  
**t\_QUAD**. Quadratic Numbers *x + y \* quadgen(D,{'w})*  
**t\_POLMOD**. Polynomials modulo *g* *Mod(f,g)*  
**t\_POL**. Polynomials *a \* x^n + ... + b*  
**t\_SER**. Power Series *f + 0(x^k)*  
**t\_RFRAC**. Rational Functions *f/g*  
**t\_QFB**. Binary quadratic form *Qfb(a,b,c)*  
**t\_VEC/t\_COL**. Row/Column Vectors *[x,y,z], [x,y,z]~*  
**t\_VEC** integer range *[1..10]*

**t\_VECSMALL**. Vector of small ints *Vecsmall([x,y,z])*  
**t\_MAT**. Matrices *[a,b;c,d]*  
**t\_LIST**. Lists *List([x,y,z])*  
**t\_STR**. Strings *"abc"*  
**t\_INFINITY**.  $±∞$  *+oo, -oo*

## Reserved Variable Names

$π ≈ 3.14, γ ≈ 0.57, C ≈ 0.91, I = √{-1}$  **Pi, Euler, Catalan, I**  
Landau's big-oh notation **O**

## Information about an Object, Precision

PARI type of object *x* *type(x)*  
length of *x* / size of *x* in memory *#x, sizebyte(x)*  
real precision / bit precision of *x* *precision(x), bitprecision(x)*  
*p*-adic, series prec. of *x* *padicprec(x,p), serprec(x,v)*  
current dynamic precision *getlocalprec, getlocalbitprec*

## Operators

basic operations *+, -, \*, /, ^, sqr*  
*i←i+1, i←i-1, i←i\*j, ...* *i++, i--, i\*=j, ...*  
Euclidean quotient, remainder *x\y, x%y, x%y, divrem(x,y)*  
shift *x* left or right *n* bits *x<<n, x>>n* or *shift(x,±n)*  
multiply by  $2^n$  *shiftmul(x,n)*  
comparison operators *<=, <, >=, >, ==, !=, ==, lex, cmp*  
boolean operators (or, and, not) *||, &&, !*  
bit operations *bitand, bitneg, bitor, bitxor, bitneg*  
maximum/minimum of *x* and *y* *max(x,y), min(x,y)*  
sign of *x* (gives  $-1, 0, 1$ ) *sign(x)*  
binary exponent of *x* *exponent(x)*  
derivative of *f*, 2nd derivative, etc. *f', f'', ...*  
differential operator *diffop(f,v,d,{n=1})*  
quote operator (formal variable) *'x*  
assignment *x = value*  
simultaneous assignment  $x ← v[1], y ← v[2]$  *[x,y] = v*

## Select Components

*Caveat*: components start at index  $n = 1$ .  
*n*-th component of *x* *component(x,n)*  
*n*-th component of vector/list *x* *x[n]*  
components *a, a + 1, ... , b* of vector *x* *x[a..b]*  
(*m, n*)-th component of matrix *x* *x[m,n]*  
row *m* or column *n* of matrix *x* *x[m,], x[,n]*  
numerator/denominator of *x* *numerator(x), denominator(x)*

## Random Numbers

random integer/prime in  $[0, N[$  *random(N), randomprime(N)*  
get/set random seed *getrand, setrand(s)*

## Conversions

to vector, matrix, vec. of small ints **Col/Vec, Mat, Vecsmall**  
to list, set, map, string **List, Set, Map, Str**  
create ( $x \bmod y$ ) **Mod(x,y)**  
make *x* a polynomial of *v* **Pol(x,{v})**  
variants of **Pol** *et al.*, in reverse order **Polrev, Vecrev, Colrev**  
make *x* a power series of *v* **Ser(x,{v})**  
convert *x* to simplest possible type **simplify(x)**  
object *x* with real precision *n* **precision(x,n)**  
object *x* with bit precision *n* **bitprecision(x,n)**  
set precision to *p* digits in dynamic scope **localprec(p)**  
set precision to *p* bits in dynamic scope **localbitprec(p)**

## Character strings

convert to TeX representation **strtex(x)**  
string from bytes / from format+args **strchr, sprintf**  
split string / join strings **strsplit, strjoin**  
convert time *t* ms. to h, m, s, ms format **strtime(t)**

## Conjugates and Lifts

conjugate of a number *x* **conj(x)**  
norm of *x*, product with conjugate **norm(x)**  
 $L^p$  norm of *x* ( $L^∞$  if no *p*) **normlp(x,{p})**  
square of  $L^2$  norm of *x* **norml2(x)**  
lift of *x* from Mods and *p*-adics **lift, centerlift(x)**  
recursive lift **liftall**  
lift all **t\_INT** and **t\_PADIC** ( $→t\_INT$ ) **liftint**  
lift all **t\_POLMOD** ( $→t\_POL$ ) **lifttpol**

## Lists, Sets & Maps

**Sets** (= row vector with strictly increasing entries w.r.t. **cmp**)  
intersection of sets *x* and *y* **setintersect(x,y)**  
set of elements in *x* not belonging to *y* **setminus(x,y)**  
symmetric difference  $xΔy$  **setdelta(x,y)**  
union of sets *x* and *y* **setunion(x,y)**  
does *y* belong to the set *x* **setsearch(x,y,{flag})**  
set of all  $f(x,y), x ∈ X, y ∈ Y$  **setbinop(f,X,Y)**  
is *x* a set? **setisset(x)**

**Lists**. create empty list:  $L = \text{List}()$

append *x* to list *L* **listput(L,x,{i})**  
remove *i*-th component from list *L* **listpop(L,{i})**  
insert *x* in list *L* at position *i* **listinsert(L,x,i)**  
sort the list *L* in place **listsort(L,{flag})**

**Maps**. create empty dictionary:  $M = \text{Map}()$

attach value *v* to key *k* **mapput(M,k,v)**  
recover value attach to key *k* or error **mapget(M,k)**  
is key *k* in the dict? (set *v* to  $M(k)$ ) **mapisdefined(M,k,{&v})**  
evaluate *f* at  $M(k)$  **mapapply(M,k,f)**  
remove *k* from map domain **mapdelete(M,k)**

## GP Programming

### User functions and closures

*x, y* are formal parameters; *y* defaults to **Pi** if parameter omitted;  
*z, t* are local variables (lexical scope), *z* initialized to 1.

**fun(x, y=Pi) = my(z=1, t); seq**  
**fun = (x, y=Pi) -> my(z=1, t); seq**

attach help message *h* to *s* **addhelp(s,h)**  
undefine symbol *s* (also kills help) **kill(s)**

**Control Statements** (*X*: formal parameter in expression *seq*)  
if  $a ≠ 0$ , evaluate *seq<sub>1</sub>*, else *seq<sub>2</sub>* **if(a,{seq1},{seq2})**

eval. *seq* for  $a ≤ X ≤ b$  **for(X = a, b, seq)**  
... for  $X ∈ v$  **foreach(v, X, seq)**  
... for primes  $a ≤ X ≤ b$  **forprime(X = a, b, seq)**  
... for primes  $≡ a \pmod q$  **forprimestep(X = a, b, q, seq)**  
... for composites  $a ≤ X ≤ b$  **forcomposite(X = a, b, seq)**  
... for  $a ≤ X ≤ b$  stepping *s* **forstep(X = a, b, s, seq)**  
... for *X* dividing *n* **fordiv(n, X, seq)**  
...  $X = [n, factor(n)], a ≤ n ≤ b$  **forfactored(X = a, b, seq)**  
... as above, *n* squarefree **forsquarefree(X = a, b, seq)**  
...  $X = [d, factor(d)], d | n$  **fordivfactored(n, X, seq)**  
multivariable **for**, lex ordering **forvec(X = v, seq)**

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loop over partitions of  $n$   
... permutations of  $S$   
... subsets of  $\{1, \dots, n\}$   
...  $k$ -subsets of  $\{1, \dots, n\}$   
... vectors  $v, q(v) \leq B; q > 0$   
...  $H < G$  finite abelian group  
evaluate  $seq$  until  $a \neq 0$   
while  $a \neq 0$ , evaluate  $seq$   
exit  $n$  innermost enclosing loops  
start new iteration of  $n$ -th enclosing loop  
return  $x$  from current subroutine

## Exceptions, warnings

raise an exception / warning  
type of error message  $E$   
try  $seq_1$ , evaluate  $seq_2$  on error

## Functions with closure arguments / results

number of arguments of  $f$   
select from  $v$  according to  $f$   
apply  $f$  to all entries in  $v$   
evaluate  $f(a_1, \dots, a_n)$   
evaluate  $f(\dots f(f(a_1, a_2), a_3) \dots, a_n)$   
calling function as closure

## Sums & Products

sum  $X = a$  to  $X = b$ , initialized at  $x$   
sum entries of vector  $v$   
product of all vector entries  
sum  $expr$  over divisors of  $n$   
... assuming  $expr$  multiplicative  
product  $a \leq X \leq b$ , initialized at  $x$   
product over primes  $a \leq X \leq b$

## Sorting

sort  $x$  by  $k$ -th component  
min.  $m$  of  $x$  ( $m = x[i]$ ), max.  
does  $y$  belong to  $x$ , sorted wrt.  $f$   
 $\prod g^x \rightarrow$  factorization ( $\Rightarrow$  sorted, unique  $g$ )

## Input/Output

print with/without  $\backslash n$ , TeX format  
pretty print matrix  
print fields with separator  
formatted printing  
write  $args$  to file  
write  $x$  in binary format  
read file into GP  
... return as vector of lines  
... return as vector of strings  
read a string from keyboard

## Files and file descriptors

File descriptors allow efficient small consecutive reads or writes from or to a given file. The argument  $n$  below is always a descriptor, attached to a file in **r**(ead), **w**(rite) or **a**(ppend) mode.  
get descriptor  $n$  for file  $path$  in given  $mode$   
... from shell  $cmd$  output (pipe)

close descriptor  
commit pending write operations  
read logical line from file  
... raw line from file  
write  $s \backslash n$  to file  
... write  $s$  to file

forpart( $p = n, seq$ )  
forperm( $S, p, seq$ )  
forsubset( $n, p, seq$ )  
forsubset( $[n, k], p, seq$ )  
forqfvec( $v, q, b, seq$ )  
forsubgroup( $H = G$ )  
until( $a, seq$ )  
while( $a, seq$ )  
break( $\{n\}$ )  
next( $\{n\}$ )  
return( $\{x\}$ )

error(), warning()  
errname( $E$ )  
iferr( $seq_1, E, seq_2$ )

arity( $f$ )  
select( $f, v$ )  
apply( $f, v$ )  
call( $f, a$ )  
fold( $f, a$ )  
self()

sum( $X = a, b, expr, \{x\}$ )  
vecsum( $v$ )  
vecprod( $v$ )  
sumdiv( $n, X, expr$ )  
sumdivmult( $n, X, expr$ )  
prod( $X = a, b, expr, \{x\}$ )  
prodeuler( $X = a, b, expr$ )

vecsort( $x, \{k\}, \{fl = 0\}$ )  
vecmin( $x, \{\&i\}$ ), vecmax  
vecsearch( $x, y, \{f\}$ )  
matreduce( $m$ )

print, print1, printtex  
printp  
printsep( $sep, \dots$ ), printsep1  
printf()  
write, write1, writetex( $file, args$ )  
writebin( $file, x$ )  
read( $\{file\}$ )  
readvec( $\{file\}$ )  
readstr( $\{file\}$ )  
input()

fileopen( $path, mode$ )  
fileextern( $cmd$ )  
fileclose( $n$ )  
fileflush( $n$ )  
fileread( $n$ )  
filereadstr( $n$ )  
filewrite( $n, s$ )  
filewrite1( $n, s$ )

## Timers

CPU time in  $ms$  and reset timer  
CPU time in  $ms$  since gp startup  
time in  $ms$  since UNIX Epoch  
timeout command after  $s$  seconds

## Interface with system

allocates a new stack of  $s$  bytes  
alias  $old$  to  $new$   
install function from library  
execute system command  $a$   
... and feed result to GP  
... returning GP string  
get \$VAR from environment  
expand env. variable in string

## Parallel evaluation

These functions evaluate their arguments in parallel (pthreads or MPI); args. must not access global variables (use **export** for this) and must be free of side effects. Enabled if threading engine is not *single* in gp header.

evaluate  $f$  on  $x[1], \dots, x[n]$   
evaluate closures  $f[1], \dots, f[n]$   
as **select**  
as **sum**  
as **vector**  
eval  $f$  for  $i = a, \dots, b$   
... for each element  $x$  in  $v$   
... for  $p$  prime in  $[a, b]$   
... for  $p = a \bmod q$   
... for  $i = a, a + s, \dots, b$   
... multivariate  
export  $x$  to parallel world  
... all dynamic variables  
frees exported value  $x$   
... all exported values

## Linear Algebra

dimensions of matrix  $x$   
multiply two matrices  
... assuming result is diagonal  
concatenation of  $x$  and  $y$   
extract components of  $x$   
transpose of vector or matrix  $x$   
adjoint of the matrix  $x$   
eigenvectors/values of matrix  $x$   
characteristic/minimal polynomial of  $x$   
trace/determinant of matrix  $x$   
permanent of matrix  $x$   
Frobenius form of  $x$   
QR decomposition  
apply **matqr**'s transform to  $v$

## Constructors & Special Matrices

$\{g(x) : x \in v \text{ s.t. } f(x)\}$   
 $\{x : x \in v \text{ s.t. } f(x)\}$   
 $\{g(x) : x \in v\}$   
row vec. of  $expr$  eval'ed at  $1 \leq i \leq n$   
col. vec. of  $expr$  eval'ed at  $1 \leq i \leq n$   
vector of small ints

gettime()  
getabstime()  
getwalltime()  
alarm( $s, expr$ )  
allocatemem( $\{s\}$ )  
alias( $new, old$ )  
install( $f, code, \{gpf\}, \{lib\}$ )  
system( $a$ )  
extern( $a$ )  
externstr( $a$ )  
getenv("VAR")  
strexpend( $x$ )

parapply( $f, x$ )  
pareval( $f$ )  
parselect( $f, A, \{flag\}$ )  
parsum( $i = a, b, expr$ )  
parvector( $n, i, \{expr\}$ )  
parfor( $i = a, \{b\}, f, \{r\}, \{f_2\}$ )  
parforeach( $v, x, f, \{r\}, \{f_2\}$ )  
parforprime( $p = a, \{b\}, f, \{r\}, \{f_2\}$ )  
parforprimestep( $p = a, \{b\}, q, f, \{r\}, \{f_2\}$ )  
parforstep( $i = a, \{b\}, s, f, \{r\}, \{f_2\}$ )  
parforvec( $X = v, f, \{r\}, \{f_2\}, \{flag\}$ )  
export( $x$ )  
exportall()  
unexport( $x$ )  
unexportall()

matsize( $x$ )  
 $x * y$   
matmultodiagonal( $x, y$ )  
concat( $x, \{y\}$ )  
vecextract( $x, y, \{z\}$ )  
 $x\sim$ , mattranspose( $x$ )  
matadjoint( $x$ )  
mateigen( $x$ )  
charpoly( $x$ ), minpoly( $x$ )  
trace( $x$ ), matdet( $x$ )  
matpermanent( $x$ )  
matfrobenius( $x$ )  
matqr( $x$ )  
mathouseholder( $Q, v$ )

$[g(x) \mid x \leftarrow v, f(x)]$   
 $[x \mid x \leftarrow v, f(x)]$   
 $[g(x) \mid x \leftarrow v]$   
vector( $n, \{i\}, \{expr\}$ )  
vectorv( $n, \{i\}, \{expr\}$ )  
vectorsmall( $n, \{i\}, \{expr\}$ )

$[c, c \cdot x, \dots, c \cdot x^n]$   
 $[1, 2^x, \dots, n^x]$   
matrix  $1 \leq i \leq m, 1 \leq j \leq n$   
define matrix by blocks  
diagonal matrix with diagonal  $x$   
is  $x$  diagonal?  
 $x \cdot \text{matdiagonal}(d)$   
 $n \times n$  identity matrix

Hessenberg form of square matrix  $x$   
 $n \times n$  Hilbert matrix  $H_{ij} = (i + j - 1)^{-1}$   
 $n \times n$  Pascal triangle  
companion matrix to polynomial  $x$   
Sylvester matrix of  $x$  and  $y$

## Gaussian elimination

kernel of matrix  $x$   
intersection of column spaces of  $x$  and  $y$   
solve  $MX = B$  ( $M$  invertible)  
one sol of  $M * X = B$   
basis for image of matrix  $x$   
columns of  $x$  *not* in **matimage**  
supplement columns of  $x$  to get basis  
rows, cols to extract invertible matrix  
rank of the matrix  $x$   
solve  $MX = B \bmod D$   
image mod  $D$   
kernel mod  $D$   
inverse mod  $D$   
determinant mod  $D$

## Lattices & Quadratic Forms

### Quadratic forms

evaluate  ${}^t x Q y$   
evaluate  ${}^t x Q x$   
signature of quad form  ${}^t y * x * y$   
decomp into squares of  ${}^t y * x * y$   
eigenvalues/vectors for real symmetric  $x$   
Cholesky decomposition of  $x$

### HNF and SNF

upper triangular Hermite Normal Form  
HNF of  $x$  where  $d$  is a multiple of  $\det(x)$   
multiple of  $\det(x)$   
HNF of  $(x \mid \text{diagonal}(D))$   
elementary divisors of  $x$   
 $q$ -rank from elementary divisors  
elementary divisors of  $\mathbf{Z}[a]/(f'(a))$   
integer kernel of  $x$   
**Z**-module  $\leftrightarrow$  **Q**-vector space

### Lattices

LLL-algorithm applied to columns of  $x$   
... for Gram matrix of lattice  
find up to  $m$  sols of  $qfeval(x, y) \leq b$   
... up to  $m$  closest vectors to  $t$   
 $v, v[i] :=$  number of  $y$  s.t.  $qfeval(x, y) = i$   
perfection rank of  $x$   
find isomorphism between  $q$  and  $Q$

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Send comments and corrections to (Karim.Belabas@math.u-bordeaux.fr)

powers( $x, n, \{c = 1\}$ )  
dirpowers( $n, x$ )  
matrix( $m, n, \{i\}, \{j\}, \{expr\}$ )  
matconcat( $B$ )  
matdiagonal( $x$ )  
matisdiagonal( $x$ )  
matmultiagonal( $x, d$ )  
matid( $n$ )  
mathess( $x$ )  
mathilbert( $n$ )  
matpascal( $n - 1$ )  
matcompanion( $x$ )  
polsylvestermatrix( $x, y$ )

matker( $x, \{flag\}$ )  
matintersect( $x, y$ )  
matsolve( $M, B$ )  
matinverseimage( $M, B$ )  
matimage( $x$ )  
matimagecompl( $x$ )  
matsupplement( $x$ )  
matindexrank( $x$ )  
matrank( $x$ )  
matsolvenmod( $M, D, B$ )  
matimagemod( $M, D$ )  
matkermod( $M, D$ )  
matinvmod( $M, D$ )  
matdetmod( $M, D$ )

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precompute for isomorphism test with  $q$  `qfisominit( $q$ )`  
 automorphism group of  $q$  `qfauto( $q$ )`  
 convert `qfauto` for GAP/Magma `qfautoexport( $G, \{flag\}$ )`  
 orbits of  $V$  under  $G \subset \text{GL}(V)$  `qforbits( $G, V$ )`

## Polynomials & Rational Functions

all defined polynomial variables `variables()`  
 get var. of highest priority (higher than  $v$ ) `varhigher( $name, \{v\}$ )`  
 ... of lowest priority (lower than  $v$ ) `varlower( $name, \{v\}$ )`

### Coefficients, variables and basic operators

degree of  $f$  `poldegree( $f$ )`  
 coef. of degree  $n$  of  $f$ , leading coef. `polcoef( $f, n$ ), pollead`  
 main variable / all variables in  $f$  `variable( $f$ ), variables( $f$ )`  
 replace  $x$  by  $y$  in  $f$  `subst( $f, x, y$ )`  
 evaluate  $f$  replacing vars by their value `eval( $f$ )`  
 replace polynomial expr.  $T(x)$  by  $y$  in  $f$  `substpol( $f, T, y$ )`  
 replace  $x_1, \dots, x_n$  by  $y_1, \dots, y_n$  in  $f$  `substvec( $f, x, y$ )`  
 $f \in A[x]$ ; reciprocal polynomial  $x^{\deg f} f\left(\frac{1}{x}\right)$  `polrecip( $f$ )`  
 gcd of coefficients of  $f$  `content( $f$ )`  
 derivative of  $f$  w.r.t.  $x$  `deriv( $f, \{x\}$ )`  
 ...  $n$ -th derivative of  $f$  `derivn( $f, n, \{x\}$ )`  
 formal integral of  $f$  w.r.t.  $x$  `intformal( $f, \{x\}$ )`  
 formal sum of  $f$  w.r.t.  $x$  `sumformal( $f, \{x\}$ )`

### Constructors & Special Polynomials

interpolation polynomial at  $(x[1], y[1]), \dots, (x[n], y[n])$ , evaluated at  $t$ , with error estimate  $e$  `polinterpolate( $x, \{y\}, \{t\}, \{&e\}$ )`  
 monic polynomial from roots  $r$  `polfromroots( $r$ )`  
 $T_n/U_n, H_n$  `polchebyshev( $n$ ), polhermite( $n$ )`  
 $P_n, L_n^{(\alpha)}$  `pollegendre( $n$ ), pollaguerre( $n, a$ )`  
 $n$ -th cyclotomic polynomial  $\Phi_n$  `polcyclo( $n$ )`  
 return  $n$  if  $f = \Phi_n$ , else 0 `poliscyclo( $f$ )`  
 is  $f$  a product of cyclotomic polynomials? `poliscycloprod( $f$ )`  
 Zagier's polynomial of index  $(n, m)$  `polzagier( $n, m$ )`

### Resultant, elimination

discriminant of polynomial  $f$  `poldisc( $f$ )`  
 find factors of `poldisc( $f$ )` `poldiscfactors( $f$ )`  
 resultant  $R = \text{Res}_v(f, g)$  `polresultant( $f, g, \{v\}$ )`  
 $[u, v, R], xu + yv = \text{Res}_v(f, g)$  `polresultantext( $x, y, \{v\}$ )`  
 solve Thue equation  $f(x, y) = a$  `thue( $t, a, \{sol\}$ )`  
 initialize  $t$  for Thue equation solver `thueinit( $f$ )`

### Roots and Factorization (Complex/Real)

complex roots of  $f$  `polroots( $f$ )`  
 bound complex roots of  $f$  `polrootsbound( $f$ )`  
 number of real roots of  $f$  (in  $[a, b]$ ) `polsturm( $f, \{[a, b]\}$ )`  
 real roots of  $f$  (in  $[a, b]$ ) `polrootsreal( $f, \{[a, b]\}$ )`  
 complex embeddings of `t.POLMOD`  $z$  `conjvec( $z$ )`

### Roots and Factorization (Finite fields)

factor  $f$  mod  $p$ , roots `factormod( $f, p$ ), polrootsmod`  
 factor  $f$  over  $\mathbf{F}_p[x]/(T)$ , roots `factormod( $f, [T, p]$ ), polrootsmod`  
 squarefree factorization of  $f$  in  $\mathbf{F}_q[x]$  `factormodSQF( $f, \{D\}$ )`  
 distinct degree factorization of  $f$  in  $\mathbf{F}_q[x]$  `factormodDDF( $f, \{D\}$ )`  
 factor  $n$ -th cyclotomic pol.  $\Phi_n$  mod  $p$  `factormodcyclo( $n, p$ )`

### Roots and Factorization ( $p$ -adic fields)

factor  $f$  over  $\mathbf{Q}_p$ , roots `factorpadic( $f, p, r$ ), polrootspadic`  
 $p$ -adic root of  $f$  congruent to  $a$  mod  $p$  `padicappr( $f, a$ )`  
 Newton polygon of  $f$  for prime  $p$  `newtonpoly( $f, p$ )`  
 Hensel lift  $A/\text{lc}(A) = \prod_i B[i] \bmod p^e$  `polhensellift( $A, B, p, e$ )`

$T = \prod (x - z_i) \mapsto \prod (x - \omega(z_i)) \in \mathbf{Z}_p[x]$  `polteichmuller( $T, p, e$ )`  
 extensions of  $\mathbf{Q}_p$  of degree  $N$  `padicfields( $p, N$ )`  
**Roots and Factorization (Miscellaneous)**  
 symmetric powers of roots of  $f$  up to  $n$  `polysym( $f, n$ )`  
 Graeffe transform of  $f$ ,  $g(x^2) = f(x)f(-x)$  `polgraeffe( $f$ )`  
 factor  $f$  over coefficient field `factor( $f$ )`  
 cyclotomic factors of  $f \in \mathbf{Q}[X]$  `polcyclofactors( $f$ )`

### Finite Fields

A finite field is encoded by any element (`t_FFELT`).  
 find irreducible  $T \in \mathbf{F}_p[x]$ ,  $\deg T = n$  `ffinit( $p, n, \{x\}$ )`  
 Create  $t$  in  $\mathbf{F}_q \simeq \mathbf{F}_p[t]/(T)$  `t = ffgent( $T, 't$ )`  
 ... indirectly, with implicit  $T$  `t = ffgent( $q, 't$ ); T = t.mod`  
 map  $m$  from  $\mathbf{F}_q \ni a$  to  $\mathbf{F}_{q^k} \ni b$  `m = ffembed( $a, b$ )`  
 build  $K = \mathbf{F}_q[x]/(P)$  extending  $\mathbf{F}_q \ni a$ , `ffextend( $a, P$ )`  
 evaluate map  $m$  on  $x$  `ffmap( $m, x$ )`  
 inverse map of  $m$  `ffinvmmap( $m$ )`  
 compose maps  $m \circ n$  `ffcompomap( $m, n$ )`  
 $x$  as polmod over codomain of map  $m$  `ffmaprel( $m, x$ )`  
 $F^n$  over  $\mathbf{F}_q \ni a$  `fffrobenius( $a, n$ )`  
 $\#\{\text{monic irred. } T \in \mathbf{F}_q[x], \deg T = n\}$  `ffnbirred( $q, n$ )`

### Formal & $p$ -adic Series

truncate power series or  $p$ -adic number `truncate( $x$ )`  
 valuation of  $x$  at  $p$  `valuation( $x, p$ )`

### Dirichlet and Power Series

Taylor expansion around 0 of  $f$  w.r.t.  $x$  `taylor( $f, x$ )`  
 Laurent series of closure  $F$  up to  $x^k$  `laurentseries( $f, k$ )`  
 $\sum a_k b_k t^k$  from  $\sum a_k t^k$  and  $\sum b_k t^k$  `serconvol( $a, b$ )`  
 $f = \sum a_k t^k$  from  $\sum (a_k/k!) t^k$  `serlaplace( $f$ )`  
 reverse power series  $F$  so  $F(f(x)) = x$  `serreverse( $f$ )`  
 remove terms of degree  $< n$  in  $f$  `serchop( $f, n$ )`  
 Dirichlet series multiplication / division `dirmul, dirdiv( $x, y$ )`  
 Dirichlet Euler product ( $b$  terms) `direuler( $p = a, b, expr$ )`

### Transcendental and $p$ -adic Functions

real, imaginary part of  $x$  `real( $x$ ), imag( $x$ )`  
 absolute value, argument of  $x$  `abs( $x$ ), arg( $x$ )`  
 square/ $n$ th root of  $x$  `sqrt( $x$ ), sqrtn( $x, n, \{&z\}$ )`  
 all  $n$ -th roots of 1 `rootsof1( $n$ )`  
 FFT of  $[f_0, \dots, f_{n-1}]$  `w = fftinit( $n$ ), fft/fftinw( $w, f$ )`  
 trig functions `sin, cos, tan, cotan, sinc`  
 inverse trig functions `asin, acos, atan`  
 hyperbolic functions `sinh, cosh, tanh, cotanh`  
 inverse hyperbolic functions `asinh, acosh, atanh`  
 $\log(x), \log(1+x), e^x, e^x - 1$  `log, loglp, exp, expm1`  
 Euler  $\Gamma$  function,  $\log \Gamma, \Gamma'/\Gamma$  `gamma, lngamma, psi`  
 half-integer gamma function  $\Gamma(n+1/2)$  `gammah( $n$ )`  
 Riemann's zeta  $\zeta(s) = \sum n^{-s}$  `zeta( $s$ )`  
 $\sum_{1 \leq n \leq N} n^s$  `dirpowersum( $N, s$ )`  
 Hurwitz's  $\zeta(s, x) = \sum (n+x)^{-s}$  `zetahurwitz( $s, x$ )`  
 Lerch  $\Phi(z, s, x) = \sum z^n (n+x)^{-s}$  `lerchphi( $z, s, x$ )`  
 Lerch  $L(s, x, t) = \Phi(e^{2i\pi t}, s, x)$  `lerchzeta( $s, x, t$ )`  
 multiple zeta value (MZV),  $\zeta(s_1, \dots, s_k)$  `zetamult( $s, \{T\}$ )`  
 all MZVs for weight  $\sum s_i = n$  `zetamultall( $n$ )`  
 convert MZV id to  $[s_1, \dots, s_k]$  `zetamultconvert( $f, \{flag\}$ )`  
 MZV dual sequence `zetamultdual( $s$ )`  
 multiple polylog  $Li_{s_1, \dots, s_k}(z_1, \dots, z_k)$  `polylogmult( $s, z$ )`

incomplete  $\Gamma$  function ( $y = \Gamma(s)$ ) `incgam( $s, x, \{y\}$ )`  
 complementary incomplete  $\Gamma$  `incgamc( $s, x$ )`  
 $\int_x^\infty e^{-t} dt/t, (2/\sqrt{\pi}) \int_x^\infty e^{-t^2} dt$  `eint1, erfc`  
 elliptic integral of 1st and 2nd kind `ellK( $k$ ), ellE( $k$ )`  
 dilogarithm of  $x$  `dilog( $x$ )`  
 $m$ -th polylogarithm of  $x$  `polylog( $m, x, \{flag\}$ )`  
 $U$ -confluent hypergeometric function `hyperu( $a, b, u$ )`  
 Hypergeometric  ${}_pF_q(A, B; z)$  `hypergeom( $A, B, z$ )`  
 Bessel  $J_n(x), J_{n+1/2}(x)$  `besselj( $n, x$ ), besseljh( $n, x$ )`  
 Bessel  $I_\nu, K_\nu, H_\nu^1, H_\nu^2, Y_\nu$  `(bessel)i, k, h1, h2, y`  
 $k$ -th zero of  $J_\nu(x)$  `besseljzero( $nu, \{k = 1\}$ )`  
 $k$ -th zero of  $Y_\nu(x)$  `besselyzero( $nu, \{k = 1\}$ )`  
 Airy functions  $A_i(x), B_i(x)$  `airy( $x$ )`  
 Lambert  $W: x$  s.t.  $xe^x = y$  `lambertw( $y$ )`  
 Teichmuller character of  $p$ -adic  $x$  `teichmuller( $x$ )`

## Iterations, Sums & Products

### Numerical integration for meromorphic functions

Behaviour at endpoint for Double Exponential (DE) methods: either a scalar ( $a \in \mathbf{C}$ , regular) or  $\pm\infty$  (decreasing at least as  $x^{-2}$ ) or

|                                       |  |
|---------------------------------------|--|
| $(x-a)^{-\alpha}$ singularity         | <code>[<math>a, a</math>]</code>                                     |
| exponential decrease $e^{-\alpha x }$ | <code>[<math>\pm\infty, a</math>], <math>\alpha &gt; 0</math></code> |
| slow decrease $ x ^\alpha$            | <code>... <math>\alpha &lt; -1</math></code>                         |
| oscillating as $\cos(kx)$             | <code><math>\alpha = kI, k &gt; 0</math></code>                      |
| oscillating as $\sin(kx)$             | <code><math>\alpha = -kI, k &gt; 0</math></code>                     |

numerical integration `intnum( $x = a, b, f, \{T\}$ )`  
 weights  $T$  for intnum `intnuminit( $a, b, \{m\}$ )`  
 weights  $T$  incl. kernel  $K$  `intfuncinit( $t = a, b, K, \{m\}$ )`  
 integrate  $(2i\pi)^{-1} f$  on circle  $|z-a| = R$  `intcirc( $x = a, R, f, \{T\}$ )`

### Other integration methods

$n$ -point Gauss-Legendre `intnumgauss( $x = a, b, f, \{n\}$ )`  
 weights for  $n$ -point Gauss-Legendre `intnumgaussinit( $\{n\}$ )`  
 quasi-periodic function, period  $2H$  `intnumosc( $x = a, f, H$ )`  
 Romberg (low accuracy) `intnumromb( $x = a, b, f, \{flag\}$ )`

### Numerical summation

sum of series  $f(n), n \geq a$  (low accuracy) `suminf( $n = a, expr$ )`  
 sum of alternating/positive series `sumalt, sumpos`  
 sum of series using Euler-Maclaurin `sumnum( $n = a, f, \{T\}$ )`  
 ... Sidi summation `sumnumsidi( $n = a, f$ )`  
 $\sum_{n \geq a} F(n)$ ,  $F$  rational function `sumnumrat( $F, a$ )`  
 $\dots \sum_{p \geq a} F(p^s)$  `sumeulerrat( $F, \{s = 1\}, \{a = 2\}$ )`  
 weights for `sumnum`,  $a$  as in DE `sumnuminit( $\{\infty, a\}$ )`  
 sum of series by Monien summation `sumnummonien( $n = a, f, \{T\}$ )`  
 weights for `sumnummonien` `sumnummonieninit( $\{\infty, a\}$ )`  
 sum of series using Abel-Plana `sumnumap( $n = a, f, \{T\}$ )`  
 weights for `sumnumap`,  $a$  as in DE `sumnumapinit( $\{\infty, a\}$ )`  
 sum of series using Lagrange `sumnumlagrange( $n = a, f, \{T\}$ )`  
 weights for `sumnumlagrange` `sumnumlagrangeinit`

### Products

product  $a \leq X \leq b$ , initialized at  $x$  `prod( $X = a, b, expr, \{x\}$ )`  
 product over primes  $a \leq X \leq b$  `prodeuler( $X = a, b, expr$ )`  
 infinite product  $a \leq X \leq \infty$  `prodinf( $X = a, expr$ )`  
 $\prod_{n \geq a} F(n)$ ,  $F$  rational function `prodnumrat( $F, a$ )`  
 $\prod_{p \geq a} F(p^s)$  `prodeulerrat( $F, \{s = 1\}, \{a = 2\}$ )`

